Industrial Use Case

Mobile Systems

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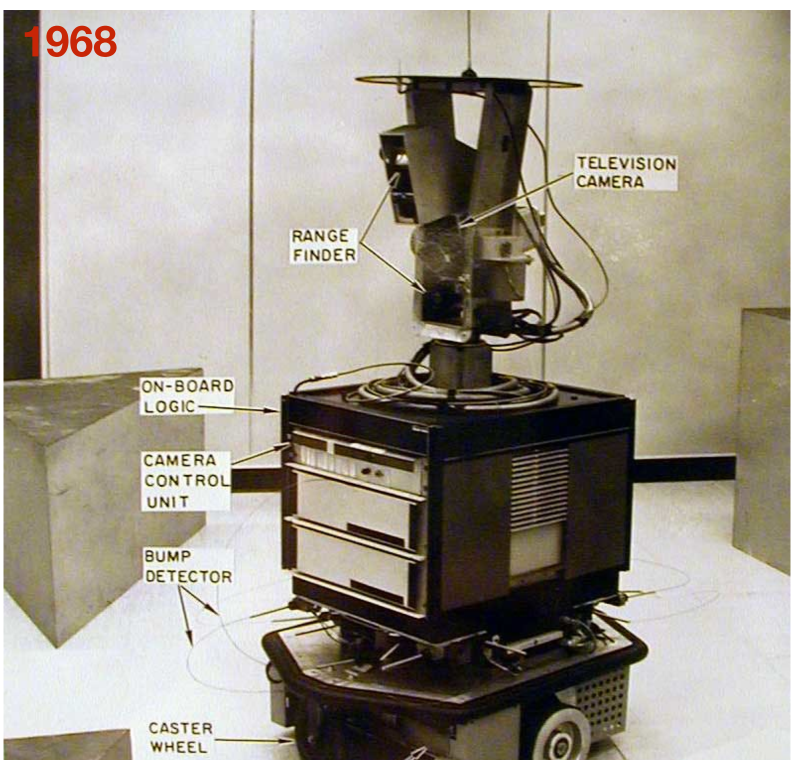
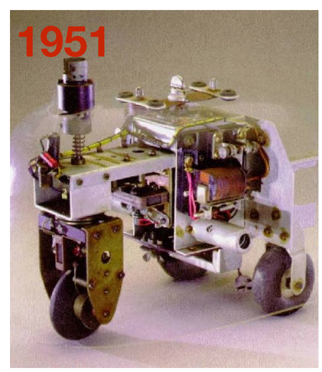
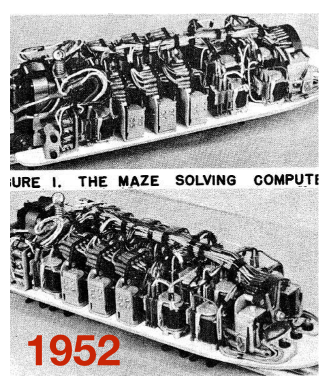
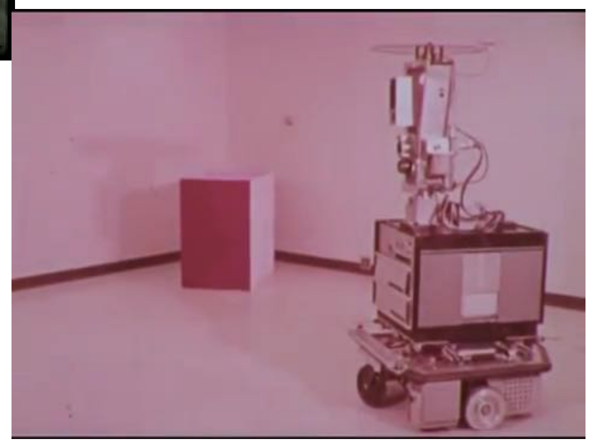




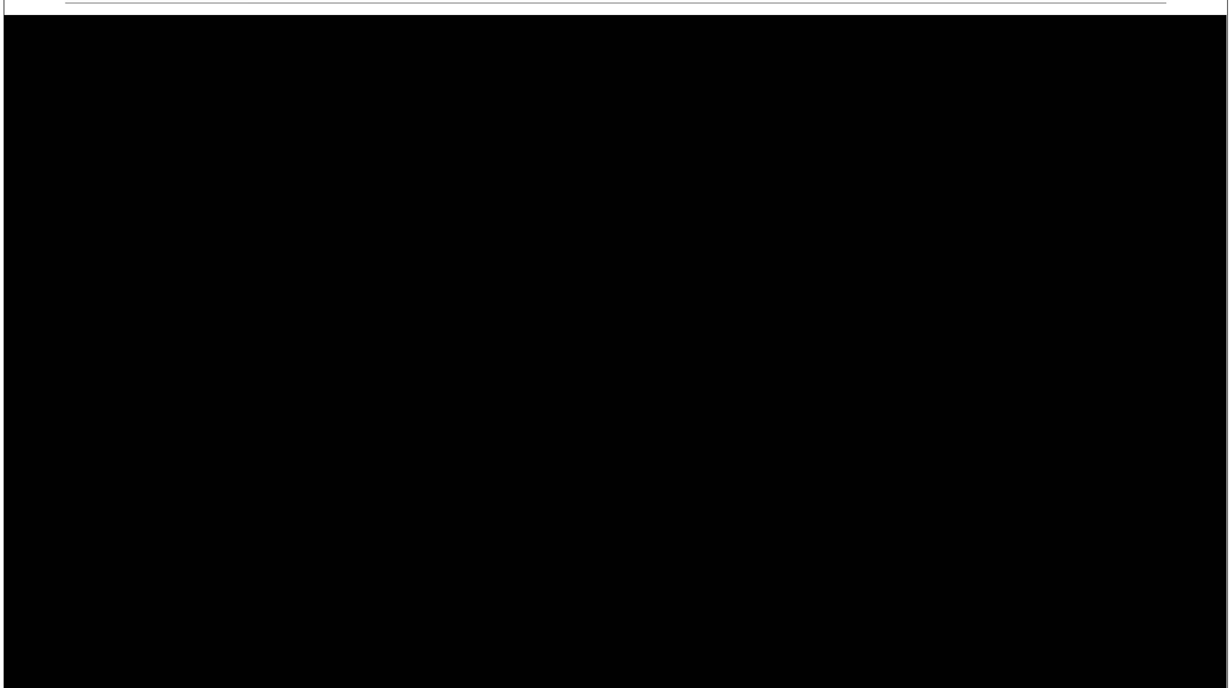




More Modern AGVs

A widely advertised example - KIVA Systems (now Amazon)

KIVA example



Other Modalities

Mobility: Train

- Configuration: 1D
- Task Space: R
- C = T

Mobility: Hovercraft

- Configuration:(x,y,θ)
- Actuators: 2DOF
- Task Space: SE(2)
- C = T

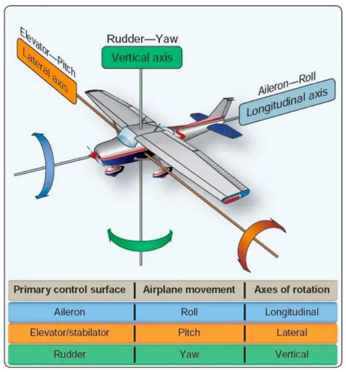
Mobility: Helicopter

- Configuration: (x,y,z,θ_r,θ_p,θ_y)
- Actuators: 4DOF (thrust,pitch,roll,tail)
- Task Space: SE(3)
- C = T

Boeing A160 Hummingbird

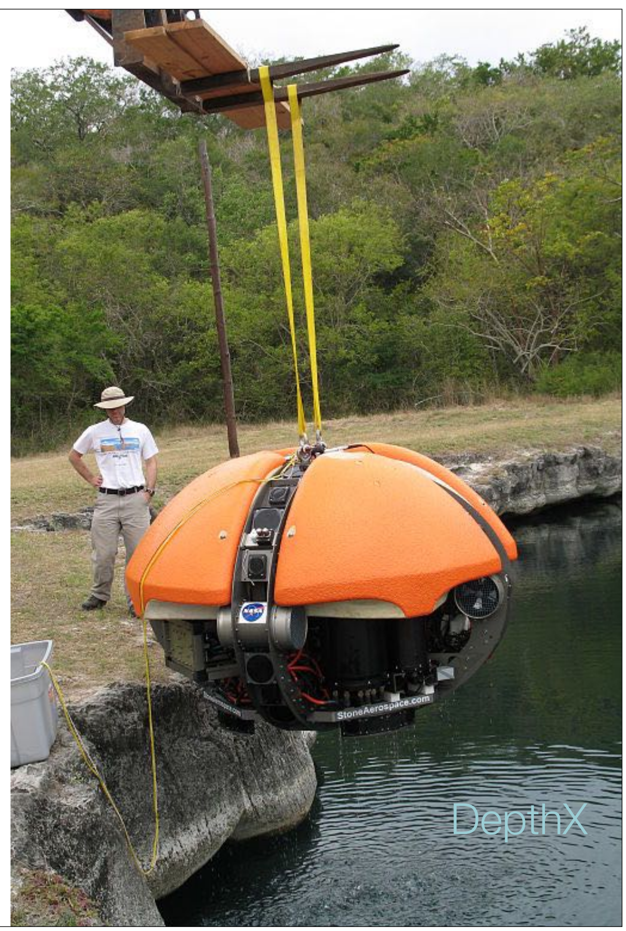
Mobility: Fixed WING

- Configuration: $(x,y,z,\theta_r,\theta_p,\theta_y)$
- Actuators: 4DOF (thrust, ail, elev, rud)
- Task Space: SE(3)
- C = T



Mobility: Submersible

- Configuration: 6D
- Fully Actuated
- Task Space: SE(3)
- C = T



Wheels

- Regular wheel
  - Non-holonomic constraint
  - x'=v, y'=0
- Omnidirectional wheel
  - No such constraint

Mobility Recap

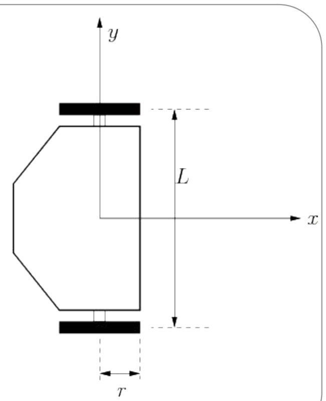
Table 4.1. Summary of parameters for three different types of vehicle. The +g notation indicates that the gravity field can be considered as an extra actuator

Vehicle	Degrees of freedom	Number of actuators	Fully actuated?
Train	1	1	✓
Hovercraft	3	2	×
Helicopter	6	4+g	×
Fixed wing aircraft	6	4+g	×
6-thruster AUV	6	6	✓
Car	3	2	×

2 Kinematic Models

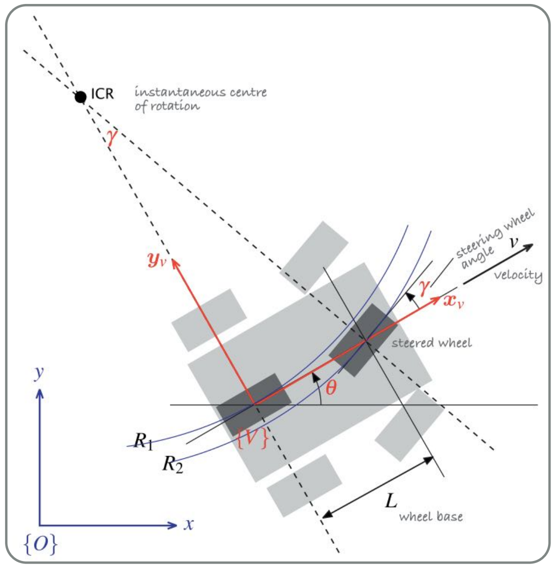
Differential Drive


(a)

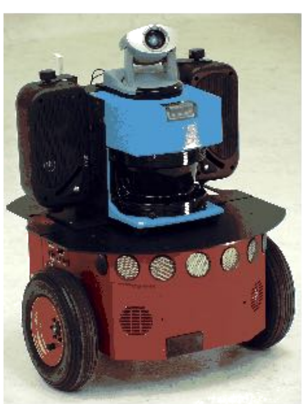


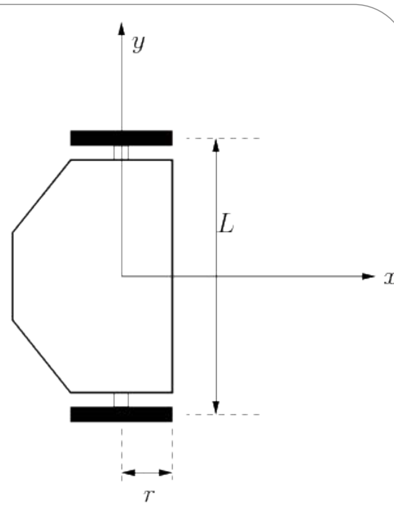
(b)

Bicycle Model



Differential drive


(a)



(b)

$$\dot{\xi}_R = \begin{bmatrix} \frac{r}{2}(\dot{\varphi}_R + \dot{\varphi}_L) \\ 0 \\ \frac{r}{L}(\dot{\varphi}_R - \dot{\varphi}_L) \end{bmatrix}$$

(this is in the body frame)

Forward only:

$$\dot{\varphi}_R = \frac{\mathrm{v_x}}{\mathrm{r}}$$

$$\dot{\varphi}_L = \frac{\mathrm{v_x}}{\mathrm{r}}$$

Rotation only:

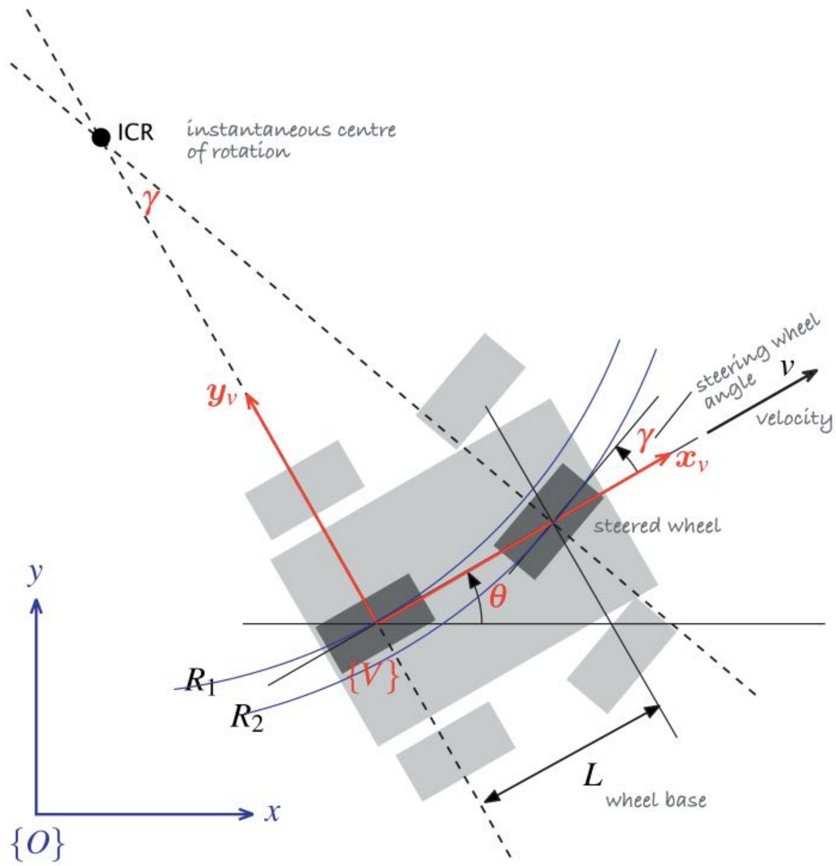
$$\dot{\varphi}_R = \frac{\omega \mathrm{L}}{2\mathrm{r}}$$

$$\dot{\varphi}_L = -\frac{\omega L}{2r}$$

Both:

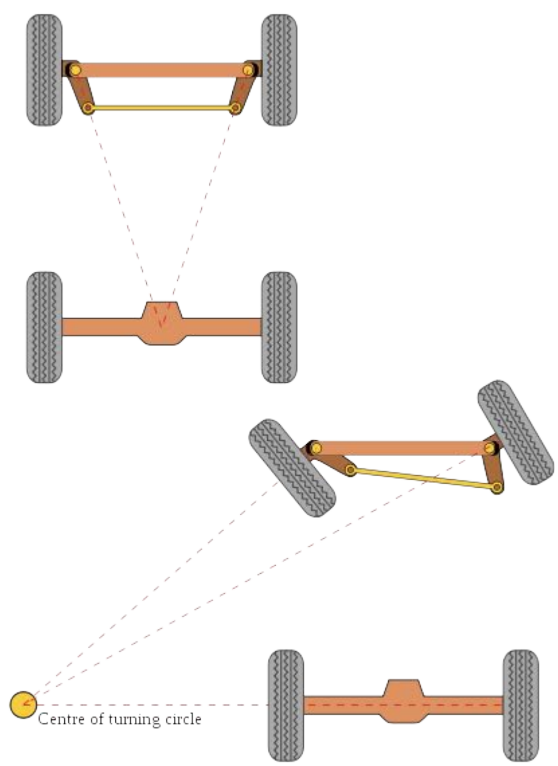
$$\dot{\varphi}_R = \frac{\omega \mathrm{L}}{2\mathrm{r}} + \frac{\mathrm{v_x}}{\mathrm{r}}$$

$$\dot{\varphi}_L = -\frac{\omega L}{2r} + \frac{\mathrm{v_x}}{\mathrm{r}}$$



Ackerman Steering

- Four-wheeled vehicle
- L and R move on circular paths of different radius
- 1812 patent



Equations of Motion
(See book)

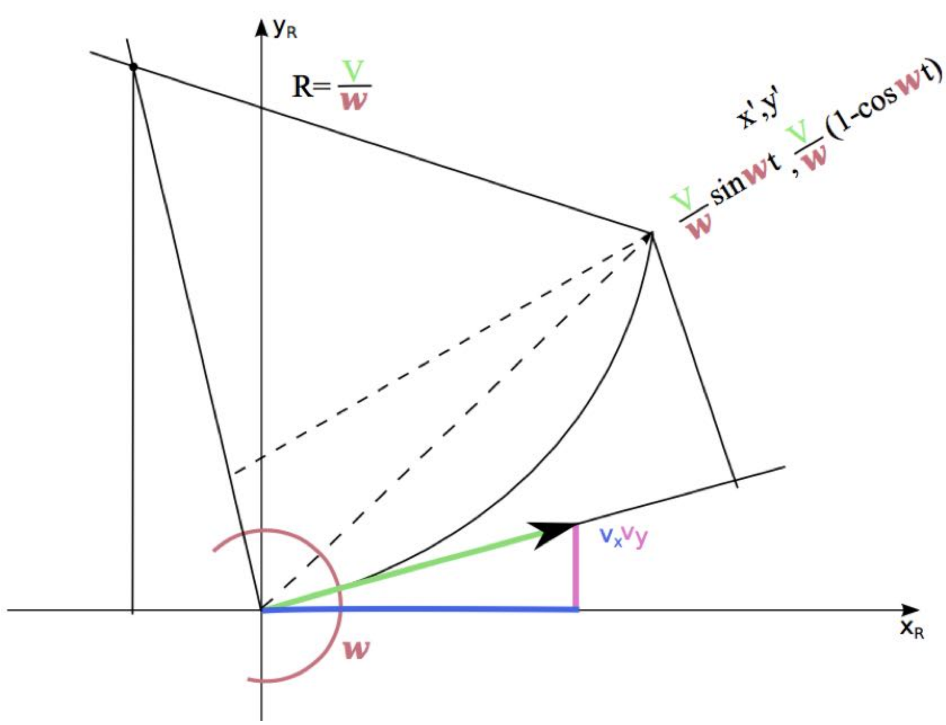
- Angular velocity
  ~ steering angle γ
- Translation velocity
  ~ velocity v
- Non-holonomic constraint
- Undefined for 90° angle

$$\dot{x} = v\cos\theta$$
$$\dot{y} = v\sin\theta$$
$$\dot{\theta} = \frac{v}{L}\tan\gamma$$

$$\dot{y}\cos\theta - \dot{x}\sin\theta \equiv 0$$

(this is in the world frame)

2D Twist



2D Twist

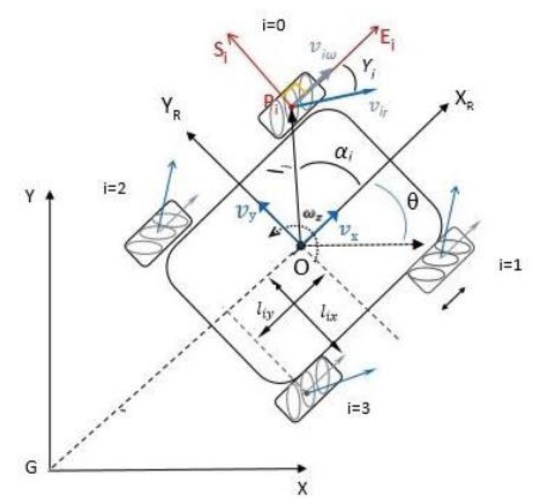
A 2D twist is simply the derivative of a 2D rigid transformation

$$\dot{\xi}_R^W = \begin{bmatrix} \dot{t}_x \\ \dot{t}_y \\ \dot{\theta} \end{bmatrix}$$

A robot undergoing a constant twist, expressed in the robot frame, traces out a circular trajectory with radius $R = v/\omega$. Starting from the origin, after some time T we obtain

$$\xi(T) = \left(\begin{bmatrix} \cos\omega T & -\sin\omega T \\ \sin\omega T & \cos\omega T \end{bmatrix}, \frac{1}{\omega} \begin{bmatrix} 1-\cos\omega T & \sin\omega T \\ -\sin\omega T & 1-\cos\omega T \end{bmatrix} \begin{bmatrix} -v_y \\ v_x \end{bmatrix} \right)$$

Mechanum wheels



Kinematic Model of a Four Mecanum Wheeled Mobile Robot

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College of Automation

The general kinematic model

$$\begin{bmatrix}\omega_1\\ \omega_2\\ \omega_3\\ \omega_4\end{bmatrix} = \frac{-1}{r}\begin{bmatrix}\frac{\cos(\beta_1-\gamma_1)}{\sin\gamma_1} & \frac{\sin(\beta_1-\gamma_1)}{\sin\gamma_1} & \frac{l_1\sin(\beta_1-\gamma_1-\alpha_1)}{\sin\gamma_1}\\ \frac{\cos(\beta_2-\gamma_2)}{\sin\gamma_2} & \frac{\sin(\beta_2-\gamma_2)}{\sin\gamma_2} & \frac{l_2\sin(\beta_2-\gamma_2-\alpha_2)}{\sin\gamma_2}\\ \frac{\cos(\beta_3-\gamma_3)}{\sin\gamma_3} & \frac{\sin(\beta_3-\gamma_3)}{\sin\gamma_3} & \frac{l_3\sin(\beta_3-\gamma_3-\alpha_3)}{\sin\gamma_3}\\ \frac{\cos(\beta_4-\gamma_4)}{\sin\gamma_4} & \frac{\sin(\beta_4-\gamma_4)}{\sin\gamma_4} & \frac{l_4\sin(\beta_4-\gamma_4-\alpha_4)}{\sin\gamma_4}\end{bmatrix}\begin{bmatrix}v_X\\ v_Y\\ \omega_z\end{bmatrix}.$$

Basic kinematic model

$$\begin{bmatrix}v_x\\ v_y\\ \omega_z\end{bmatrix} = \frac{r}{4}\begin{bmatrix}1 & 1 & 1 & 1\\ -1 & 1 & 1 & -1\\ -\frac{1}{(l_x+l_y)} & \frac{1}{(l_x+l_y)} & -\frac{1}{(l_x+l_y)} & \frac{1}{(l_x+l_y)}\end{bmatrix}\begin{bmatrix}\omega_1\\ \omega_2\\ \omega_3\\ \omega_4\end{bmatrix}$$

i	Wheels	α_i	β_i	γ_i	l_i	l_{ix}	l_{iy}
0	1sw	$\pi/4$	$\pi/2$	$-\pi/4$	l	l_x	l_y
1	2sw	$-\pi/4$	$-\pi/2$	$\pi/4$	l	l_x	l_y
2	3sw	$3\pi/4$	$\pi/2$	$\pi/4$	l	l_x	l_y
3	4sw	$-3\pi/4$	$-\pi/2$	$-\pi/4$	l	l_x	l_y

https://research.ijcaonline.org/volume113/number3/pxc3901586.pdf

Motion of omnidirectional platform

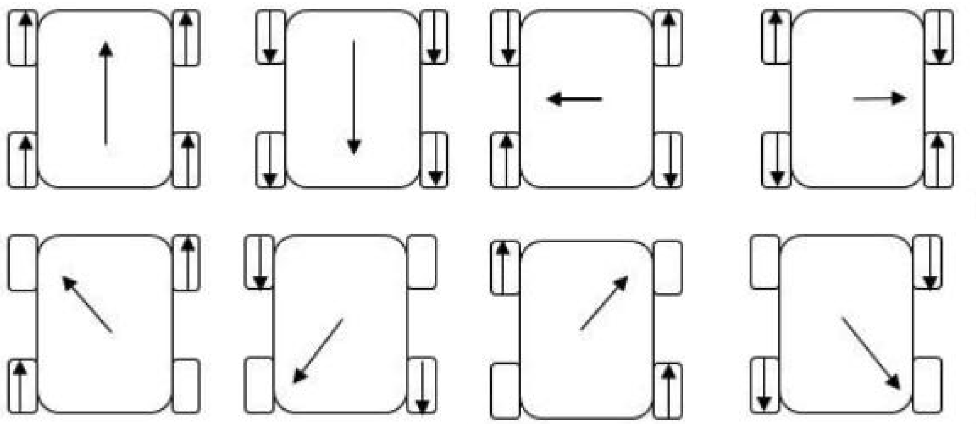
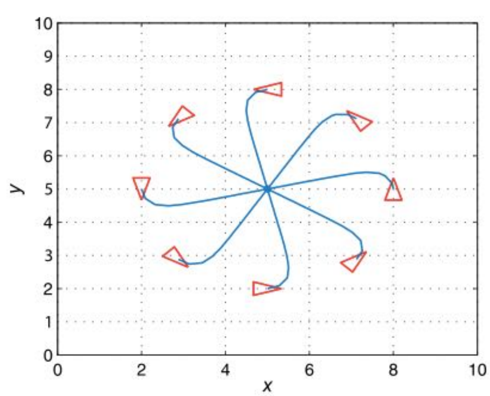


Fig 4: Motions of Omnidirectional platform

Moving to a Point

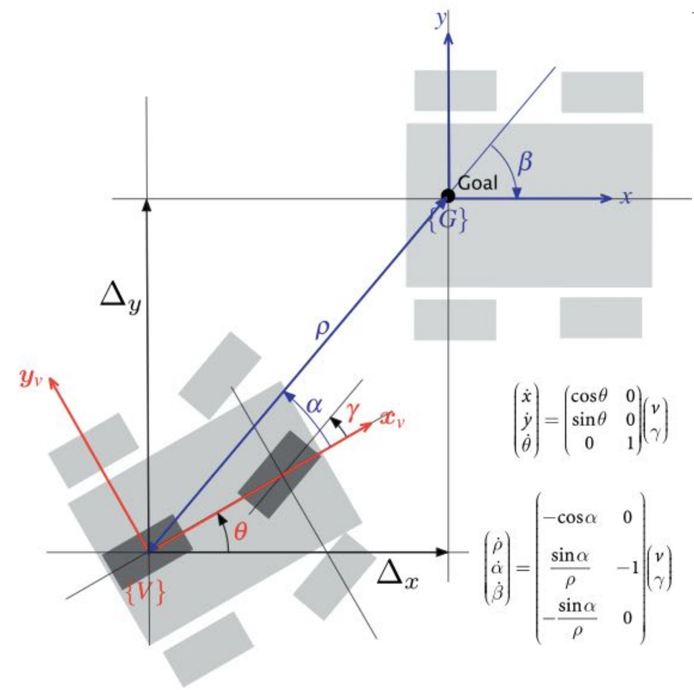


$$v^* = K_v\sqrt{(x^* - x)^2 + (y^* - y)^2}$$

$$\theta^* = \tan^{-1}\frac{y^* - y}{x^* - x}$$

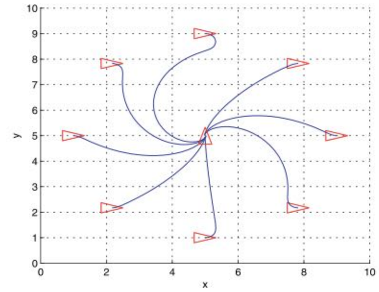
$$\gamma = K_h(\theta^* \ominus \theta),\ \ K_h > 0$$

Moving to a Pose



$$\begin{pmatrix}\dot{x}\\ \dot{y}\\ \dot{\theta}\end{pmatrix} = \begin{pmatrix}\cos\theta & 0\\ \sin\theta & 0\\ 0 & 1\end{pmatrix}\begin{pmatrix}v\\ \gamma\end{pmatrix}$$

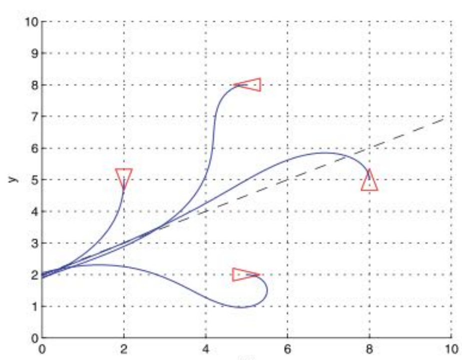
$$\begin{pmatrix}\dot{\rho}\\ \dot{\alpha}\\ \dot{\beta}\end{pmatrix} = \begin{pmatrix}-\cos\alpha & 0\\ \frac{\sin\alpha}{\rho} & -1\\ -\frac{\sin\alpha}{\rho} & 0\end{pmatrix}\begin{pmatrix}v\\ \gamma\end{pmatrix}$$



$$v = k_\rho \rho$$
$$\gamma = k_\alpha \alpha + k_\beta \beta$$

$$\begin{pmatrix}\dot{\rho}\\ \dot{\alpha}\\ \dot{\beta}\end{pmatrix} = \begin{pmatrix}-k_\rho \cos\alpha\\ k_\rho \sin\alpha - k_\alpha \alpha - k_\beta \beta\\ -k_\rho \sin\alpha\end{pmatrix}$$

Following a Line

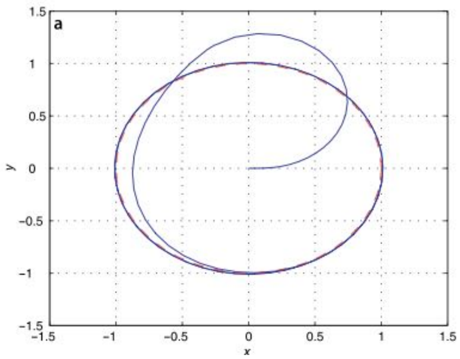


$$d = \frac{(a,b,c)\cdot(x,y,1)}{\sqrt{a^2+b^2}}$$
$$\alpha_d = -K_d d,\ K_d > 0$$

$$\theta^* = \tan^{-1}\frac{-a}{b}$$
$$\alpha_h = K_h(\theta^* \ominus \theta),\ K_h > 0$$

$$\gamma = -K_d d + K_h(\theta^* \ominus \theta)$$

Following a path



$$e = \sqrt{(x^* - x)^2 + (y^* - y)^2} - d^*$$

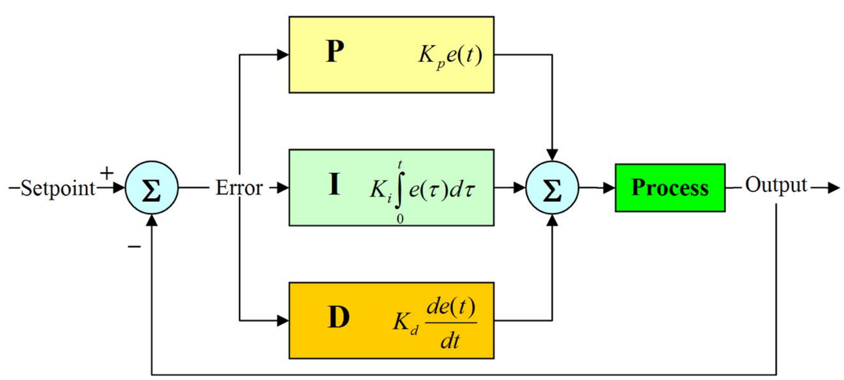
$$v^* = K_v e + K_i \int e\,dt$$

$$\theta^* = \tan^{-1} \frac{y^* - y}{x^* - x}$$

$$\alpha = K_h(\theta^* \ominus \theta), \quad K_h > 0$$

PID Control

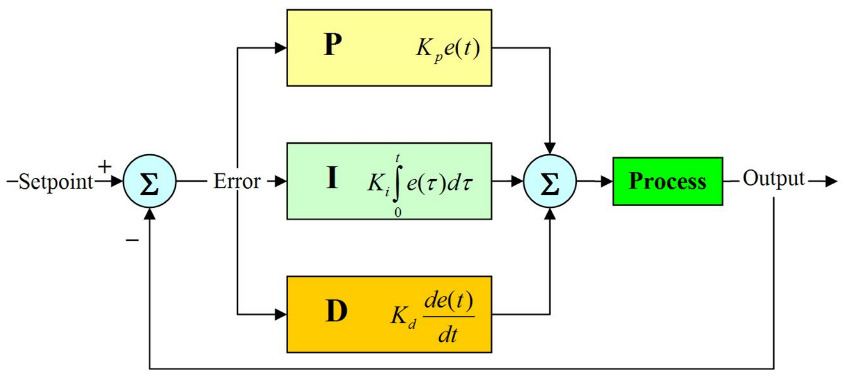
- **P** = Proportional
- **I** = Integral
- **D** = Derivative



- Simple
- Robust, excellent results in many cases
- Controls 95% of all industrial processes

PID Control

- **P** = Proportional: correct
- **I** = Integral: reduce tracking error
- **D** = Derivative: stabilize (anticipate)



Code Example

```
class PID:
    def __init__(self, Kp, Ki, Kd):
        self.previous_error = 0
        self.integral = 0
        self.Kp = Kp
        self.Ki = Ki
        self.Kd = Kd

    def calc(self, dt, setpoint, y):
        error = setpoint - y
        self.integral += error*dt
        derivative = (error - self.previous_error)/dt
        u = self.Kp*error + self.Ki*self.integral + self.Kd*derivative
        self.previous_error = error
        return u
```

Python example due to Andy Henshaw at GTRI

PID Tuning

- Ziegler-Nichols
  - Start with P only
  - Increase Kp until output oscillates
  - Call that "Ultimate gain" Ku ,and call the oscillation period Tu

Ziegler–Nichols method[1]			
Control Type	K_p	K_i	K_d
P	$0.5K_u$	-	-
PI	$0.45K_u$	$1.2K_p/T_u$	-
PD	$0.8K_u$	-	$K_pT_u/8$
classic PID[2]	$0.60K_u$	$2K_p/T_u$	$K_pT_u/8$

Source: Wikipedia

Summary

- Configuration of mobile systems
  - Configuration Space
  - Actuators
  - 2D vs 3D space
  - Is the system fully controlled
- Simple example of kinematic models for mobile systems
  - Bi-cycle, Differential drive, Ackerman,
- Basic control algorithms